Case 1: Testing a Claim About a Mean $\mu$: Large Sample (N > 30)

A. REJECTION REGION METHOD

1. **State the Null Hypothesis $H_0$**
   This is a hypothesis that must contain a statement of equality, such as $\leq$, $=,$ or $\geq$

2. **State the Alternative Hypothesis $H_a$ (some texts use $H_1$)**
   This is the complement of the null hypothesis; i.e., it must be true if $H_0$ is false.
   This hypothesis must contain a statement of inequality, such as $<$, $\neq$, or $>$.  

3. **Specify the Level of Significance Alpha ($\alpha$)**
   The probability of rejecting the null hypothesis when it is true.

4. **Decide whether the test is left-tailed, right-tailed, or two-tailed.**
   If the alternative hypothesis contains:
   - $<$ it is a left-tailed test
   - $\neq$ it is a two-tailed test
   - $>$ it is a right-tailed test

   **HINT:** The inequality symbols `$<$ and `$>' point to the rejection region.

5. **Find the Critical Value(s)**
   This is the value that separates the rejection region from the non-rejection region.
   It is a $z$ score that for $\alpha$
   - left-tailed test is the **negative** $z$ score that corresponds to an area of $\alpha$
   - right-tailed test is the **positive** $z$ score that corresponds to an area of $1 - \alpha$
   - two-tailed test are the two $z$ scores that correspond to the areas of $\frac{1}{2} \alpha$ and $1 - \frac{1}{2} \alpha$

### SOME COMMON CRITICAL Z SCORES

<table>
<thead>
<tr>
<th>Alpha ($\alpha$)</th>
<th>Tails</th>
<th>Critical Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>2</td>
<td>$+1.96$, $-1.96$</td>
</tr>
<tr>
<td></td>
<td>LEFT</td>
<td>$-1.645$</td>
</tr>
<tr>
<td></td>
<td>RIGHT</td>
<td>$+1.645$</td>
</tr>
<tr>
<td>.01</td>
<td>2</td>
<td>$+2.58$, $-2.58$</td>
</tr>
<tr>
<td></td>
<td>LEFT</td>
<td>$-2.33$</td>
</tr>
<tr>
<td></td>
<td>RIGHT</td>
<td>$+2.33$</td>
</tr>
</tbody>
</table>

*Testing a Claim About a Mean $\mu$: (Large Sample)*
6. **Sketch the Normal Distribution**
   Add the critical value(s), and shade in the corresponding rejection region. Below is an example for a two-tailed test.

   ![Normal Distribution with Critical Values](image)

7. **Calculate the Test Statistic**
   \[ Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \]
   *If \( \sigma \) is unknown, use \( s \) as an approximation

   Determine where \( Z \) is located in relation to the rejection region identified in the previous step.

8. **Make a Decision**
   If \( Z \) is in the rejection region, make a decision to ‘Reject \( H_0 \)’. If \( Z \) is outside the rejection region, make a decision to ‘Fail to Reject \( H_0 \)’.

9. **Restate the Decision in Non-Technical Terms**
   *If the original claim contained the condition of equality \((=, \leq, \geq)\), and:*
     a. you failed to reject \( H_0 \), it should be stated similar to the following: “There is not sufficient evidence to warrant rejection of the claim that … (original claim)”
     b. you rejected \( H_0 \), it should be stated similar to the following “There is sufficient evidence to warrant rejection of the claim that … (original claim)”

   *If the original claim contained the condition of inequality \((\neq, <, >)\), and:*
     a. you rejected \( H_0 \), it should be stated similar to the following: “The sample data support the claim that … (original claim).”
     b. you failed to reject \( H_0 \), it should be stated similar to the following “There is not sufficient sample evidence to support the claim that … (original claim)”
B. **P-VALUE METHOD**

The steps to conduct the hypothesis test for the mean using $P$-values are similar to the traditional method previously discussed, with the main difference that we will be comparing our $P$-value to the level of significance rather than comparing a test statistic to a rejection region. The test statistic is still required, however, since it determines the size of the tail, which in turns determines the value of $P$.

1. **State the Null Hypothesis $H_0$**
2. **State the Alternative Hypothesis $H_a$ (some texts use $H_1$)**
3. **Specify the level of significance Alpha ($\alpha$)**
4. **Decide whether the test is left-tailed, right-tailed, or two-tailed.**
5. **Calculate the Test Statistic ($Use Z because n>30$)**

\[
Z = \frac{\bar{x} - \mu}{\sigma} \quad \text{if } \sigma \text{ is unknown, then use } s \text{ as an approximation of } \sigma
\]

6. **Find the $P$-value**
   - For a left-tailed test: $P = \text{Area to the left of the test statistic}$
   - For a right-tailed test: $P = \text{Area to the right of the test statistic}$
   - For a two-tailed test: $P = \text{twice the area in the tail beyond the test statistic.}$

7. **Make a decision to Reject or Fail to Reject $H_0$**
   Reject $H_0$ if the $P$-value $\leq \alpha$. Otherwise, fail to reject $H_0$

8. **Restate the Decision in Non-Technical Terms**
   Refer to Step 9 of the previous section (Rejection Region Method) for acceptable verbiage.
C. TI-83/84 METHOD

1. Determine the Null Hypothesis $H_0$, Alternative Hypothesis $H_a$, and Significance Level $\alpha$

2. Press STATS

3. Select TESTS, then choose first option of Z-Test

   a) Input: Data (Use if your data resides in lists on the calculator) or Stats (Use if you just have summary stats such as $\sigma$ and $\mu$)
   b) Enter Null Hypothesis value $\mu_0$
   c) Enter $\sigma$

   Remember: If $\sigma$ is unknown (as it usually is), and the sample size is large, use the value of $s$ as a reasonable approximation of $\sigma$

   d) Enter sample mean $\bar{x}$ and sample size $n$
   e) Enter alternative hypothesis ($\mu \neq \mu_0$; $\mu <\mu_0$; $\mu >\mu_0$)

f) Press Calculate

Interpretation from above TI-83/84 screen shots:

1. **P test:** $P = .5481$
   If $\alpha$ is .05, then $P > \alpha$, which means do not reject the null.

2. **Rejection Region Method:** For two-tailed test, the critical values are ±1.96 significance level of .05. Since $z = -0.6007$ is between -1.96 and +1.96 (zero is in the middle), it is not in the rejection region, so do not reject the null.
Sample Exercise

Test the claim that $\mu = 40$ given a sample of $n = 75$ for which $\bar{x} = 39.2$ and $s = 3.23$. Use a significance level of .05

<table>
<thead>
<tr>
<th>Rejection Region Method</th>
<th>P- Value Method</th>
<th>TI-83/84 Method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Null: $\mu = 40$</td>
<td>Null: $\mu = 40$</td>
<td>Null: $\mu = 40$</td>
</tr>
<tr>
<td><strong>Step 2:</strong> Alternative: $\mu \neq 40$</td>
<td>Alternative: $\mu \neq 40$</td>
<td>Alternative: $\mu \neq 40$</td>
</tr>
<tr>
<td><strong>Step 3:</strong> $\alpha = .05$</td>
<td>$\alpha = .05$</td>
<td>$\alpha = .05$</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Critical Values: $Z = \pm 1.96$</td>
<td>Test Statistic $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$</td>
<td>Press STATS Select TESTS, then choose first option Z-Test</td>
</tr>
<tr>
<td></td>
<td>$= \frac{39.2 - 40.0}{3.23/\sqrt{75}}$</td>
<td>Input: Stats $\mu_0 = 40$ $\sigma = 3.23$ $\bar{x} = 39.2$ $n = 75$ $\mu \neq \mu_0$</td>
</tr>
<tr>
<td></td>
<td>$= -2.145$</td>
<td>Calculate Draw</td>
</tr>
</tbody>
</table>
| **Step 5:** Test Statistic $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ | For a two-tailed test, $P$ is twice the area in the tail of the test statistic. $Z = -2.145$ corresponds to a tail of 0.0158. Therefore $P = 2(0.0158) = 0.0316$ | Press ‘Calculate’:
| | | $Z$-Test Inp:Data Stats $\mu_0 = 40$ $\sigma = 3.23$ $\bar{x} = 39.2$ $n = 75$ $\mu \neq \mu_0$ |
| **Step 6:** Since $Z = -2.145$ is in the rejection region ($<-1.96$; see Step 4), the decision is to reject the null. | Since $P < \alpha (.0316 < .050)$, the decision is to reject the null. | Reject the null for one of two reasons:
| | | 1. $P < .05 (.032 < .05)$
| | | 2. $Z = -2.145$, which is less than -1.96 (therefore it is in the rejection region)