Case 2: Testing a Claim About a Mean $\mu$: Small Sample (N<30)

A. **REJECTION REGION METHOD**

1. **State the Null Hypothesis $H_0$**
   - This is a hypothesis that must contain a statement of equality, such as $\leq$, $=$, or $\geq$.

2. **State the Alternative Hypothesis $H_a$ (some texts use $H_1$)**
   - This is the complement of the null hypothesis; i.e., it must be true if $H_0$ is false.
   - This hypothesis must contain a statement of inequality, such as $<$, $\neq$, or $>$.  

3. **Specify the Level of Significance Alpha ($\alpha$)**
   - The probability of rejecting the null hypothesis when it is true.

4. **Find the degrees of freedom ($df=N-1$)**

5. **Decide whether the test is left-tailed, right-tailed, or two-tailed.**
   - If the alternative hypothesis contains:
     - $<$ it is a left-tailed test
     - $\neq$ it is a two-tailed test
     - $>$ it is a right-tailed test
   
   **HINT:** The inequality symbols ‘$<$’ and ‘$>$’ point to the rejection region.

6. **Find the Critical Value(s)**
   - This is the value that separates the rejection region from the non-rejection region.
   - **Using a 't' distribution table**, go to the row with $n-1$ degrees of freedom, and then go over to the column heading that contains both the type of tail and the level of significance $\alpha$. **That number is the critical value.**
   - If the hypothesis test is left-tailed, make the critical value negative; if right-tailed, positive. If the test is two-tailed, create duplicate critical values, one positive and one negative.

7. **Sketch the Normal Distribution**
   - Add the critical value(s), and shade in the corresponding rejection region. To the right is an example of a two-tailed test.
8. Calculate the Test Statistic*

\[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \]

Determine where \( t \) is located in relation to the rejection region identified in the previous step.

*If \( \sigma \) is known, and the underlying distribution is normal (or assumed normal), then use the normal distribution with:
\[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \]
proceeding as if there was a large sample >30.

Though this case is relatively rare, the student should be aware of its impact.

9. Make a Decision

If \( t \) is in the rejection region, make a decision to ‘Reject \( H_0 \)’. If \( t \) is outside the rejection region, make a decision to ‘Fail to Reject \( H_0 \)’.

10. Restate the Decision in Non-Technical Terms

If the original claim contained the condition of equality (\( =, \leq, \geq \)), and:

a. you failed to reject \( H_0 \), it should be stated similar to the following: “There is not sufficient evidence to warrant rejection of the claim that … (original claim)”

b. you rejected \( H_0 \), it should be stated similar to the following “There is sufficient evidence to warrant rejection of the claim that … (original claim)”

If the original claim contained the condition of inequality (\( \neq, <, > \)), and:

a. you rejected \( H_0 \), it should be stated similar to the following: “The sample data support the claim that … (original claim).”

b. you failed to reject \( H_0 \), it should be stated similar to the following “There is not sufficient sample evidence to support the claim that … (original claim)”
B. **P-VALUE METHOD**

Unlike the Z distribution table, the t distribution table will not give us a P-value. This is because the t distribution table includes only selected values of the significance level \( \alpha \). The most practical way to find the P-value is to use the TI-83/84 method as described in the next section.

C. **TI-83/84 METHOD**

1. **Determine the Null Hypothesis \( H_0 \), Alternative Hypothesis \( H_a \), and Significance Level \( \alpha \).**

2. **Determine critical values from t distribution table**
   
   Example: \( \alpha = .05; \mu_0 = 64.8, n=12; \bar{x} = 59.8; s=8.7, df = n - 1 = 11 \)

   From the table, using \( df = 11, \alpha = .05 \), two-tailed test yields critical values of -2.201 and +2.201

3. **Press STATS**

4. **Select TESTS, then choose second option of T-Test**
   
   a) Input: **Data** (Use if your data resides in lists on the calculator) or **Stats** (Use if you just have summary stats such as \( \sigma \) and \( \mu \))
   
   b) Enter Null Hypothesis value \( \mu_0 \)
   
   c) Enter sample mean \( \bar{x} \), sample standard deviation \( S_x \), and sample size \( n \)
   
   d) Enter alternative hypothesis (\( \mu \neq \mu_0; \mu < \mu_0; \mu > \mu_0 \))
   
   e) After steps a-d, press **Calculate**. Results are below.

   ![TI-83/84 Screen Shots]

   **Interpretation from above TI-83/84 screen shots:**

   1. **P test:** \( P = .072 \)
    
      If \( \alpha = .05 \), then \( P > \alpha \), which means **do not reject** the null.

   2. **Rejection Region Method:** For two-tailed test, the critical values are \( \pm 2.201 \) significance level of .05. Since \( t = -1.991 \) is between -2.201 and +2.201 (zero is in the middle), it is **not** in the rejection region, so **do not reject** the null.
Sample Exercise

Test the claim that $\mu > 165$ given a sample of $n = 7$ for which $\bar{x} = 252.7$ and $s = 27.26$. Use a significance level of .01

<table>
<thead>
<tr>
<th>Rejection Region Method</th>
<th>TI-83/84 Method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Null: $\mu \leq 165$</td>
<td>Null: $\mu \leq 165$</td>
</tr>
<tr>
<td><strong>Step 2:</strong> Alternative: $\mu &gt; 165$</td>
<td>Alternative: $\mu &gt; 165$</td>
</tr>
<tr>
<td><strong>Step 3:</strong> $\alpha = .01$</td>
<td>$\alpha = .01$</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Critical Values: From the $t$ distribution table, using $df=6$, $\alpha=.01$, and right-tailed test, the critical value is $+3.143$</td>
<td>From the $t$ distribution table, using degrees of freedom =6, $\alpha=.01$, and right-tailed test, the critical value is $+3.143$</td>
</tr>
<tr>
<td><strong>Step 5:</strong> Test Statistic</td>
<td>Press STAT, Select TESTS; choose second option T- Test; input the data; Press Calculate</td>
</tr>
<tr>
<td>$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$</td>
<td>$t = \frac{252.7 - 165}{27.6/\sqrt{7}} = 8.407$</td>
</tr>
</tbody>
</table>
| **Step 6:** Since $t= 8.407$ is in the rejection region ($> 3.143$; see Step 4), the decision is to reject the null. | Since $t= 8.407$ is in the rejection region ($> 3.143$; see Step 4), the decision is to reject the null. Also, the $P$-value is 0.000078, which is much less than $\alpha$ of 0.01. Therefore, this is a second reason to reject the null hypothesis.