Case 3: Testing a Claim About a Proportion \( \rho \)

A. REJECTION REGION METHOD

1. State the Null Hypothesis \( H_0 \)
   This is a hypothesis that must contain a statement of equality, such as \( \leq, =, \) or \( \geq \).

2. State the Alternative Hypothesis \( H_a \) (some texts use \( H_1 \))
   This is the complement of the null hypothesis; i.e., it must be true if \( H_0 \) is false.
   This hypothesis must contain a statement of inequality, such as \( <, \neq, \) or \( > \).

3. Specify the Level of Significance Alpha (\( \alpha \))
   The probability of rejecting the null hypothesis when it is true.

4. Decide whether the test is left-tailed, right-tailed, or two-tailed.
   If the alternative hypothesis contains:
   - \( < \) it is a left-tailed test
   - \( \neq \) it is a two-tailed test
   - \( > \) it is a right-tailed test

   **HINT:** The inequality symbols ‘\(<\)’ and ‘\(>\)’ point to the rejection region.

5. Find the Critical Value(s)
   This is the value that separates the rejection region from the non-rejection region.
   It is a \( z \) score that for a
   - left-tailed test is the negative \( z \) score that corresponds to an area of \( \alpha \)
   - right-tailed test is the positive \( z \) score that corresponds to an area of \( 1 - \alpha \)
   - two-tailed test corresponds to the areas of \( \frac{1}{2} \alpha \) and \( 1 - \frac{1}{2} \alpha \)

### SOME COMMON CRITICAL Z SCORES

<table>
<thead>
<tr>
<th>Alpha (( \alpha ))</th>
<th>Tails</th>
<th>Critical Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>2</td>
<td>( +1.96, -1.96 )</td>
</tr>
<tr>
<td></td>
<td>LEFT</td>
<td>( -1.645 )</td>
</tr>
<tr>
<td></td>
<td>RIGHT</td>
<td>( +1.645 )</td>
</tr>
<tr>
<td>.01</td>
<td>2</td>
<td>( +2.58, -2.58 )</td>
</tr>
<tr>
<td></td>
<td>LEFT</td>
<td>( -2.33 )</td>
</tr>
<tr>
<td></td>
<td>RIGHT</td>
<td>( +2.33 )</td>
</tr>
</tbody>
</table>

---

Document created by South Campus Library Learning Commons 03/26/10. Permission to copy and use is granted for educational use provided this copyright label is displayed.
6. **Sketch the Normal Distribution**
   Add the critical value(s), and shade in the corresponding rejection region. Below is an example of a two-tailed test.

   ![Normal Distribution Diagram](image)

7. **Calculate the Test Statistic**
   \[ z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \]

   Determine where Z is located in relation to the rejection region identified in the previous step.

8. **Make a Decision**
   If Z is in the rejection region, make a decision to ‘Reject \( H_0 \)’. If Z is outside the rejection region, make a decision to ‘Fail to Reject \( H_0 \)’.

9. **Restate the Decision in Non-Technical Terms**
   **If the original claim contained the condition of equality** (\( =, \leq, \geq \)), and:
   a. you failed to reject \( H_0 \), it should be stated similar to the following: “There is not sufficient evidence to warrant rejection of the claim that … (original claim)”
   b. you rejected \( H_0 \), it should be stated similar to the following “There is sufficient evidence to warrant rejection of the claim that … (original claim)”

   **If the original claim contained the condition of inequality** (\( \neq, <, > \)), and:
   a. you rejected \( H_0 \), it should be stated similar to the following: “The sample data support the claim that … (original claim).”
   b. you failed to reject \( H_0 \), it should be stated similar to the following “There is not sufficient sample evidence to support the claim that … (original claim)”
B. **P-VALUE METHOD**

The steps to conduct the hypothesis test for the mean using P-values are similar to the traditional method previously discussed, with the main difference that we will be comparing our P-value to the level of significance rather than comparing a test statistic to a rejection region. The test statistic is still required, however, since it determines the size of the tail, which in turns determines the value of P.

1. **State the Null Hypothesis H₀**
2. **State the Alternative Hypothesis Hₐ** (some texts use H₁)
3. **Specify the level of significance** Alpha (α)
4. **Decide whether the test is left-tailed, right-tailed, or two-tailed.**
5. **Calculate the Test Statistic** *(Use Z because n>30)*
   
   \[ z = \frac{\hat{\theta} - \theta}{\sqrt{\frac{\theta(1-\theta)}{n}}} \]

6. **Find the P-value**
   - For a left-tailed test: \( P = \text{Area to the left of the test statistic} \)
   - For a right-tailed test: \( P = \text{Area to the right of the test statistic} \)
   - For a two-tailed test: \( P = \text{twice the area in the tail beyond the test statistic.} \)

7. **Make a decision to Reject or Fail to Reject H₀**
   Reject H₀ if the P-value ≤ α. Otherwise, fail to reject H₀

8. **Restate the Decision in Non-Technical Terms**
   Refer to Step 9 of the previous section (Rejection Region Method) for acceptable verbiage.
C. TI-83/84 METHOD

1. Determine the Null Hypothesis $H_0$, Alternative Hypothesis $H_a$, and Significance Level $\alpha$

2. Press STATS

3. Select TESTS, then choose Option 5: 1-PropZTest

   a) Input: Data (Use if your data resides in lists on the calculator) or Stats (Use if you just have summary stats such as $\sigma$ and $\mu$)

   Sample Data

   b) Enter Null Hypothesis value $\rho_0$ .078
   c) Enter x (# of successes) 46
   d) Enter sample size $n$ 821
   e) Enter alternative hypothesis $\rho > \rho_0$

   ![TI-83/84 Screen Shot 1-PropZTest]

   f) Press Calculate

   ![TI-83/84 Screen Shot 1-PropZTest]

   Interpretation from above TI-83/84 screen shots:

   1. **P test:** $P = .99$
      
      If $\alpha$ is .05, then $P > .99 > \alpha$, which means **do not reject** the null.

   2. **Rejection Region Method:** For right-tailed test, the critical value is +1.645 for a significance level of .05. Since $z = -2.348 < 1.645$, it is **not** in the rejection region, so **do not reject** the null.
Sample Exercise

Sample size = 1,234; # of defectives x = 20; α = .05. Test the claim that there is a 1% error rate.

<table>
<thead>
<tr>
<th>Rejection Region Method</th>
<th>P-Value Method</th>
<th>TI-83/84 Method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong>: Null: ( \hat{p} = .01 )</td>
<td>Null: ( \hat{p} = .01 )</td>
<td>Null: ( \hat{p} = .01 )</td>
</tr>
<tr>
<td><strong>Step 2</strong>: Alternative: ( p \neq .01 )</td>
<td>Alternative: ( p \neq .01 )</td>
<td>Alternative: ( p \neq .01 )</td>
</tr>
<tr>
<td><strong>Step 3</strong>: ( \alpha = .05 )</td>
<td>( \alpha = .05 )</td>
<td>( \alpha = .05 )</td>
</tr>
</tbody>
</table>
| **Step 4**: Critical Values: \( Z = \pm 1.96 \) | Test Statistic \[
Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}
\]
| Press STATS, Select TESTS, then choose first Option 5: T-Test |
| \[
Z = \frac{0.16 - 0.01}{\sqrt{\frac{0.01 * 0.99}{1234}}} = 2.19
\] | \[
1 - \text{PropZTest}
\hat{p} = .01
x = 20
n = 1234
\] |
| **Step 5**: Test Statistic \[
Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = 2.19
\] | For a two-tailed test, \( P \) is twice the area in the tail of the test statistic. Z = +2.19 corresponds to a tail of 1-0.9857 = 0.0143. Therefore \( P = 2(0.0143) = 0.0286 \) |
| Press ‘Calculate’: |
| \[
1 - \text{PropZTest}
\hat{p} = .01
x = 20
n = 1234
\] |
| **Step 6**: Since \( Z = 2.19 \) is in the rejection region (> +1.96; see Step 4), the decision is to reject the null. | Since \( P < \alpha \); i.e., .0316 < .050, the decision is to reject the null. | Reject the null for one of two reasons: |
| Press ‘Calculate’: |
| \[
1 - \text{PropZTest}
\hat{p} = .01
z = 2.191561147
p = .028411098
1 - \text{PropZTest}
\hat{p} = .01
z = 2.191561147
p = .028411098
1 - \text{PropZTest}
\hat{p} = .01
z = 2.191561147
p = .028411098
\] | 1. \( P < .05 \) (.0284 < .05) |
| 2. \( Z = +2.19 \), which is more than +1.96 (therefore it is in the rejection region) |

Testing a Claim About a Proportion

Document created by South Campus Library Learning Commons 03/26/10. Permission to copy and use is granted for educational use provided this copyright label is displayed.