**Pre Condition**

- \( f(x) \) must be continuous on \([a, b]\).
- \( f(x) \) must be differentiable on \((a, b)\).

**Condition**

- \( f(a) = f(b) \)
  
i.e. The function \( f(x) \) has the same \( y \) values at the end points of the interval.

**Conclusion:**

- Somewhere on the interval \((a, b)\), there is a turning point \((f'(x) = 0, i.e. slope is zero)\).
- The \( x \) value(s), when \( f'(x) = 0 \), that fall in \((a, b)\) are called 'c's'.

**Meaning:**

You can predict that an ordinary function will have a turning point between two points if the \( y \) values of those points are equal.

**I.e. Given**

- \( y \)
- \([a, b]\)
- \( y \) values at \( a \) and \( b \) are equal
  
i.e. \( f(a) = f(b) \)

Then there must be some 'c' (a value for \( x \)) where \( f'(x) = 0 \) (the slope is zero).
CHECKING PRE-CONDITION
1. Look at the function, is it continuous on \([a, b]\) and differentiable on \((a, b)\)? (see rules on continuity and differentiability attached).

   If it is not continuous or differentiable for all \(x\), is the problem value for \(x\) in the open interval \((a, b)\)?

   If not differentiable for some (any) \(x\) in \((a, b)\) then STOP, Rolle's Theorem does not apply! Note that this is true for both \(f(x)\) and \(f'(x)\).

CHECKING GENERAL CONDITION
2. Find \(f(a)\) and \(f(b)\) [these are the 'y' values of the function \(f(x)\) on the end points].

   If equal ... continue.

   If not equal ... STOP! Rolle's Theorem does not apply.

FINDING "c"
3. Find \(f'(x)\), does it still look continuous and differentiable?

   Find \(f'(x) = 0\)
   Solve for \(x\).

   These \(x\) values are all the turning points of the function. You want only those that fall in \((a, b)\). Those values are your \(c\) (s).
Is your function continuous \([a, b]\) and differentiable \((a, b)\)?

These functions are continuous at every point in their domain:

- **Polynomial Functions**: \(p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0\)  \
  \{ co-efficients are real numbers \}  
  \{ exponents are positive, whole numbers \}

- **Rational Functions**: (fractions where denominator \(\neq 0\))

- **Radical Functions**: \(f(x) = \sqrt{x}\) where, if \(n\) is even then \(x \geq 0\)

**Differentiable Functions**: If a function is differentiable at some point, then it is also continuous at that point. The trick is to find out if it is differentiable!

The idea is to determine whether there are any critical points of \(f(x)\) or \(f'(x)\) fall in \((a, b)\). Look specifically for quotients whose denominators will go to zero somewhere in the interval.  
  i.e. \(f(x) = \frac{x^2}{x-3}\)  
  \{ o.k. for \((-3, 2)\) \}  
  \{ not o.k. for \((2, 5)\) \}  
  \{ o.k. for \((-5, 3)\) \}

Look for radicals whose index is even but content to negative somewhere in the interval.  
  i.e. \(f'(x) = x^{\frac{1}{2}} + \frac{x-1}{2\sqrt{x}}\)  
  \{ o.k. for \((0, \infty)\) \}  
  \{ not o.k. for \((-1, 0)\) \}