Derivatives and Graphs

PURPOSE
As we know that, the derivatives can tell us a great deal about the shape of the graph of a function. This handout is designed to help the student analyzing the graph by finding maximum or minimum values and concavity of the graph.

Critical Values of Function f :
The values of c in the domain of f where f '(c) =0 or f ' (c) does not exit ,are called the critical values of f.
The critical values of f are always partition numbers of f ‘ , but f ‘ may have partition numbers that are not critical values.

Increasing and Decreasing Functions:
For the interval (a, b)
If f ‘ (x) > 0 function f increases in that interval.
f ‘ (x) < 0 function f decreases in that interval.

First Derivative test for Local Extrema :
Let c be a critical value of f
If f ’(x) changes from negative to positive at c then f ( c ) is a local minimum.

f ’ (x) 

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x

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If \( f'(x) \) changes from positive to negative at \( c \) then \( f(c) \) is a local maximum.

\[
\begin{align*}
\text{f'(x)} & \quad \begin{array}{c}
+++
\end{array} & \quad \begin{array}{c}
-
\end{array} \\
( & \quad a & \quad c & \quad b & \quad x
\end{align*}
\]

If \( f'(x) \) does not change sign at \( c \) then \( f(c) \) is neither a local maximum nor a local minimum.

\[
\begin{align*}
\text{f'(x)} & \quad \begin{array}{c}
+++
\end{array} & \quad \begin{array}{c}
+++++
\end{array} \\
( & \quad a & \quad c & \quad b & \quad x
\end{align*}
\]

\[
\begin{align*}
\text{f'(x)} & \quad \begin{array}{c}
-
\end{array} & \quad \begin{array}{c}
-
\end{array} \\
( & \quad a & \quad c & \quad b & \quad x
\end{align*}
\]

**Inflection Points of Function f:**

The value of \( c \) in the domain of \( f \) where \( f''(c) \) does not exist are called the inflection points of \( f \).

A partition number \( c \) for \( f'' \) produces an inflection point for the graph of \( f \) only if

1. \( f''(x) \) changed sign at \( c \) and
2. \( c \) is in the domain of \( f \)
Concavity of the Function $f$:

For the interval $(a, b)$

If $f''(x) > 0$ Graph concave upward

If $f''(x) < 0$ Graph concave downward

Second Derivative Test for Local Maxima and Local Minima:

Let $c$ be a critical value for $f$

If

$f'(c) = 0 \quad f''(c) > 0 \quad f(c)$ is local minimum

$f'(c) = 0 \quad f''(c) < 0 \quad f(c)$ is local maximum

$f'(c) = 0 \quad f''(c) = 0 \quad$ Test fails

The first derivative test must be used whenever $f''(c) = 0$ or $f''(c)$ does not exist.
$f''(x) > 0 \quad \Rightarrow \quad f(x) \text{ Concave up}$

$f''(x) < 0 \quad \Rightarrow \quad f(x) \text{ concave down}$

$x = 0 \quad \text{Inflexion point}$

$f''(3) < 0 \quad \Rightarrow \quad \text{Maximum}$

$f''(3) > 0 \quad \Rightarrow \quad \text{Minimum}$