Conic Sections

Purpose
To help the student understand the major types of conic sections which are an essential part of analytical geometry, a subject critical to success in calculus.

Conic Sections
1. Circle

\[(x - h)^2 + (y - k)^2 = r^2\]

Center \((h, k)\); radius \(r\)
2. Ellipse

**HORIZONTAL**

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]

Center: \((h, k)\);
Vertices: \((h - a, k), (h + a, k)\)
Foci: \((h - c, k), (h + c, k)\)

**VERTICAL**

\[
\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1
\]

Center: \((h, k)\);
Vertices: \((h, k - a), (h, k + a)\)
Foci: \((h, k - c), (h, k + c)\)

c^2 + b^2 = a^2
3. Parabola

**HORIZONTAL**

\[(y-k)^2 = 4p(x-h)\] or \[x = a(y-k)^2 + h\] where \(a = \frac{1}{4p}\)

Vertex: \((h, k)\)

Focus: \((h + p, k)\)

Directrix: \(x = h - p\)

\(p > 0\) opens right

\(p < 0\) opens left

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**VERTICAL**

\[(x-h)^2 = 4p(y-k)\] or \[y = a(x-h)^2 + k\] where \(a = \frac{1}{4p}\)

Vertex: \((h, k)\)

Focus: \((h, k+p)\)

Directrix: \(y = k - p\)

\(p > 0\) opens up

\(p < 0\) opens down
4. Hyperbola

**HORIZONTAL**

\[
\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1
\]

Center: (h, k)
Vertices: (h – a, k), (h + a, k)
Foci: (h – c, k), (h + c, k)
Slant asymptotes: \( y = \pm \frac{b}{a} (x-h) + k \)

**VERTICAL**

\[
\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1
\]

Center: (h, k)
Vertices: (h, k – a), (h, k + a)
Foci: (h, k – c), (h, k + c)
Slant asymptotes: \( y = \pm \frac{a}{b} (x-h) + k \)

\[c^2 - b^2 = a^2\]
5. Polar Equations of Conics

Conic with one focus F at the origin (pole)

Eccentricity e: Distance point P on the conic to F = e × Distance point P to directrix D

\[ PF = e \times PD \]

\[ r = \frac{ed}{1 \pm e \cos \theta} \]

Symmetric about the x-axis

\[ r = \frac{ed}{1 \pm e \sin \theta} \]

Symmetric about the y-axis

Notes:

i) \( d = \) distance of the directrix from the pole (focus)

ii) \( ed = PF(e + 1) \)

iii) + directrix in a positive direction from the pole

- directrix in a negative direction from the pole

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<tr>
<th>ELLIPSE</th>
<th>PARABOLA</th>
<th>HYPERBOLA</th>
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<tr>
<td>( e \left( = \frac{c}{a} \right) &lt; 1 )</td>
<td>( e = 1 )</td>
<td>( e \left( = \frac{c}{a} \right) &gt; 1 )</td>
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