Solving Quadratic Equations $ax^2 + bx + c = 0$

by Completing the Square

Example 1:

$$16x^2 = 8x + 63$$

Step 1
Be sure the quadratic equation is in the form $ax^2 + bx = -c$

$$16x^2 - 8x = 63$$

Step 2
Divide both sides of the equation by the lead coefficient $a$.

$$x^2 - \frac{1}{2}x = \frac{63}{16}$$

Step 3
Form a perfect square trinomial by adding the square of half of the coefficient of the $x$ term $\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2$ to both sides.

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{63}{16} + \frac{1}{16}$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{64}{16}$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = 4$$

Step 4
Write the perfect square in factored form.

$$\left(x - \frac{1}{4}\right)^2 = 4$$

Step 5
Take the square root of both sides and solve for the two solutions of $x$.

$$\sqrt{\left(x - \frac{1}{4}\right)^2} = \pm \sqrt{4}$$

$$x - \frac{1}{4} = 2 \quad \text{and} \quad x - \frac{1}{4} = -2$$

$$x = 2 + \frac{1}{4} \quad \text{and} \quad x = -2 + \frac{1}{4}$$

$$x = \frac{9}{4} \quad \text{and} \quad x = \frac{-7}{4}$$
Example 2:

\[ 5x^2 + 30x - 3 = 0 \]

Step 1
Be sure the quadratic equation is in the form \( ax^2 + bx = -c \)

\[ 5x^2 + 30x = 3 \]

Step 2
Divide both sides of the equation by the lead coefficient \( a \).

\[ x^2 + 6x = \frac{3}{5} \]

Step 3
Form a perfect square trinomial by adding the square of half of the

coefficient of the \( x \) term \( \left( \frac{1}{2} \cdot \frac{9}{2} \right)^2 \) to both sides.

\[ x^2 + 6x + 9 = \frac{3}{5} + 9 \]

\[ x^2 + 6x + 9 = \frac{3}{5} + \frac{45}{5} \]

\[ x^2 + 6x + 9 = \frac{48}{5} \]

Step 4
Write the perfect square in factored form.

\[ (x + 3)^2 = \frac{48}{5} \]

Step 5
Take the square root of both sides and solve for the two solutions of \( x \).

\[ \sqrt{(x + 3)^2} = \pm \frac{4\sqrt{15}}{5} \]

Simplify the radical on the right hand side.

\[ \sqrt{(x + 3)^2} = \pm \frac{4\sqrt{15}}{5} \]

\[ x + 3 = \frac{4\sqrt{15}}{5} \]

\[ x + 3 = -\frac{4\sqrt{15}}{5} \]

\[ x = -3 + \frac{4\sqrt{15}}{5} \]

\[ x = -3 - \frac{4\sqrt{15}}{5} \]

\[ x = -\frac{15}{3} + \frac{4\sqrt{15}}{5} \]

\[ x = -\frac{15}{5} - \frac{4\sqrt{15}}{5} \]

\[ x = -\frac{15 + 4\sqrt{15}}{5} \]

\[ x = \frac{-15 - 4\sqrt{15}}{5} \]
Quadratic Formula

Step 1
Be sure the quadratic equation is in the form $ax^2 + bx = -c$

$$ax^2 + bx + c = 0$$
$$ax^2 + bx = -c$$

Step 2
Divide both sides of the equation by the lead coefficient $a$.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 3
Form a perfect square trinomial by adding the square of half of the coefficient of the $x$ term $\left(\frac{1}{2} \left(\frac{b}{a}\right)\right)^2$ to both sides.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 4
Write the perfect square in factored form.

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Step 5
Take the square root of both sides and solve for the two solutions of $x$.

$$x + \frac{b}{2a} = \pm \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Practice Exercises

Solve by Completing the Square

1) \( x^2 + 2x - 3 = 0 \)  
6) \( 2x^2 = 3x + 20 \)

2) \( y^2 + 3y - 4 = 0 \)  
7) \( 4v^2 + 4v - 15 = 0 \)

3) \( v^2 + 4v + 1 = 0 \)  
8) \( 4v^2 - 4v - 1 = 0 \)

4) \( y^2 + 5y + 4 = 0 \)  
9) \( 6p^2 = 5p + 4 \)

5) \( P^2 + 3P = 1 \)  
10) \( 3y - 6 = (y - 1)(y - 2) \)

Answer Key

1) -3, 1  4) -1, -4  7) \( \frac{3}{2}, \frac{-5}{2} \)  10) 2, 4
2) -4, 1  5) \( \frac{3 \pm \sqrt{17}}{2} \)  8) \( \frac{1 \pm \sqrt{2}}{2} \)
3) \( -2 \pm \sqrt{3} \)  6) \( -\frac{5}{2}, 4 \)  9) \( -\frac{1}{2}, \frac{4}{3} \)

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