Sketching Polynomial and Rational Functions

Purpose
Sketching linear functions is straightforward: since it is a straight line, you just need to points to proceed. Polynomial and rational functions, however, are much more difficult since there are curves and asymptotes involved. This handout was designed to show the student a structured approach to sketch these more complex polynomial functions.

Polynomial Functions
Polynomial functions are among the simplest expressions in algebra. They are easy to evaluate: only addition and repeated multiplication are required. A polynomial function is in the form:

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \]

This is called a polynomial function of \( x \) of degree \( n \). The number \( a_n \), the coefficient of the variable to the highest power, is called the leading coefficient. \( a_n, a_{n-1}, \ldots, a_1, a_0 \) are real numbers and \( n \) is a non-negative integer. The domain is the set of all real numbers.

To sketch the polynomial function:
1. Determine the degree of the polynomial \((n)\).
2. Determine the maximum number of turning points \((n-1)\).
3. End Behavior: Find the power function that the graph of \( f \) resembles for large values of \( x \). \( y = a_n x^n \)
4. Find the \( x \)-intercepts (zeroes) if any and determine the multiplicity of each. (Multiplicity even, touches \( x \)-axis; multiplicity odd, crosses \( x \)-axis).
5. Find the \( y \)-intercepts.
Rational Functions

A rational function is a ratio of two polynomials, and can be represented as follows:

\[ R(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_1 x + b_0} \]

To sketch the rational function:

1. Find the domain (possible input values) of the function.
2. Reduce \( R \) and find the zeroes of the denominator. Each of these zeroes will indicate the location of the vertical asymptote.
3. Locate the horizontal asymptote:
   a. If \( n < m \), then \( y=0 \)
   b. If \( n = m \), then \( y=\frac{a_n}{b_m} \)
   c. If \( n = m+1 \), then use long division to find the oblique asymptote.
   d. If \( n > m+1 \), then there is no horizontal asymptote.

\[ \text{Note:} \]
\[ \text{There is only one horizontal or oblique asymptote, or the function has none.} \]

4. Find the x-intercept if one exists. These are the zeroes of the numerator when \( R \) has been reduced to lowest terms.
5. Find the y-intercept if one exists.
6. Use other clues: Test for symmetry, plot points on intervals between critical values, etc.
Exercises

A. Analyze and sketch the following polynomial equations.

1. \( y = x^2 - 5x + 6 \)
2. \( y = 2x^4 + 2x^3 - 2x^2 - 2x \)
3. \( y = x^3 + 4x^2 - 3x - 18 \)
4. \( y = -x^3 + 8 \)
5. \( y = (x - 2)(x + 3)(x - 1)^2(x + 1) \)

B. Analyze and sketch the following rational equations.

1. \( y = \frac{x-1}{x-2} \)
2. \( y = \frac{x^2-2x}{x-2} \)
3. \( y = \frac{x-3}{3x-4} \)
4. \( y = \frac{2x+1}{2x^2+3} \)
5. \( y = \frac{x^2-x}{x-2} \)
6. \( y = \frac{x-1}{x^2-2x-3} \)
Polynomial Equation Answers

A1

A2

A3

A4

A5
Rational Equation Answers

B1

$(-\infty, 2) \cup (2, \infty)$

B2

$(-\infty, 2) \cup (2, \infty)$

B3

$(-\infty, 4/3) \cup (4/3, \infty)$

B4

$(-\infty, \infty)$

B5

$(-\infty, 2) \cup (2, \infty)$

B6

$(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

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