Objective:

This handout will help students add and subtract proper fractions.

Vocabulary Review:

*Fraction* - a number that represents part of a whole. A fraction is made up of a numerator and a denominator and is written like this, \( \frac{\text{numerator}}{\text{denominator}} \). The numerator is the number above the fraction bar and the denominator is the number below the fraction bar. The denominator tells how many equal parts are in the whole. The numerator refers to how many parts out of the whole are being considered (slices eaten, $$ spent, sticks left over, etc.).

*Proper Fraction* - a fraction whose numerator is less than its denominator.

*Improper Fraction* - a fraction whose numerator is greater than or equal to its denominator.

*Like Fractions* - fractions with the same denominators.

Example:

\[
\frac{2}{5}, \frac{1}{5}
\]

*Unlike Fractions* - fractions with different denominators.

Example:

\[
\frac{2}{3}, \frac{1}{6}
\]

*Lowest Common Multiple* - the smallest non-zero number that is a multiple of two or more numbers. The LCM is the smallest positive number that other numbers will divide into without a remainder.

*Lowest Common Denominator* - the smallest whole number that is divisible by each denominator of two or more fractions. The Lowest Common Denominator (LCD) is the smallest multiple that the denominators of other fractions have in common. The LCD can never be equal to zero because the fraction will be undefined if the denominator is equal to zero.

*Equivalent Fractions* - fractions that have the same value or represent the same part of a whole, but have different denominators.
Lowest Common Multiple

To find the lowest common multiple (LCM) of two numbers, list the multiples of each number. The smallest number that is common to both lists is the LCM.

Example 1: Find the lowest common multiple of 3 and 4.

\[
\begin{align*}
3 & \rightarrow 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 \\
4 & \rightarrow 4, 8, 12, 16, 20, 24, 28, 32 \\
\end{align*}
\]

12 is the lowest common multiple of the numbers 3 and 4

Example 2: Find the lowest common multiple of 7 and 14.

\[
\begin{align*}
7 & \rightarrow 7, 14, 21, 28, 35, 42, 49, 56, 63 \\
14 & \rightarrow 14, 28, 42, 56 \\
\end{align*}
\]

14 is the lowest common multiple of the number 7 and 14

Lowest Common Denominator

Mathematically the Lowest Common Multiple and the Lowest Common Denominator are the same thing. They are always equal. LCD is used with fractions. LCM can be applied to many circumstances.

Example 3: Find the lowest common denominator of \( \frac{2}{5} \) and \( \frac{1}{3} \).

The denominators 5 and 3 have a lowest common multiple of 15. The lowest common denominator will be 15.

Equivalent Fractions

Every fraction can be represented by an infinite number of equivalent fractions. Equivalent fractions have the same overall value even if the parts look different. All equivalent fractions represent the same part of a whole. Below is an example of three equivalent fractions. Each diagram represents the same value (the shaded area).
The diagram shows that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

To create an equivalent fraction you can multiply or divide the numerator and the denominator by the same number.

**Example 4:** Find an equivalent fraction for $\frac{3}{5}$.

$$\frac{3 \cdot 3}{5 \cdot 3} = \frac{9}{15} \Rightarrow \frac{3}{5} = \frac{9}{15}$$

**Example 5:** Find an equivalent fraction for $\frac{25}{60}$.

$$\frac{25 \div 5}{60 \div 5} = \frac{5}{12} \Rightarrow \frac{25}{60} = \frac{5}{12}$$

**Adding and Subtracting Fractions with the Same Denominators**

**Step 1:** Verify that the fractions have the same denominator.

**Step 2:** Add or subtract the numerators.

**Step 3:** Simplify the fraction if necessary.

**Example 6:**

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

Both fractions have the same denominator. Add the numerators and keep the same denominator.
Example 7: \[
\frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}
\]

Both fractions have the same denominator. Subtract the numerators and keep the same denominator. Simplify the fraction by dividing the numerator and denominator by the common factor of 2. Once the Greatest Common Factor is “1”, the fraction is simplified.

Try these exercises:

1) \[
\frac{1}{6} + \frac{3}{6}
\]

2) \[
\frac{7}{15} + \frac{5}{15}
\]

3) \[
\frac{5}{7} - \frac{2}{7}
\]

4) \[
\frac{5}{8} - \frac{1}{8}
\]

5) \[
\frac{15}{16} - \frac{7}{16}
\]

Answers:

1) \[
\frac{4}{6} = \frac{2}{3}
\]

2) \[
\frac{12}{15} = \frac{4}{5}
\]

3) \[
\frac{3}{7}
\]

4) \[
\frac{4}{8} = \frac{1}{2}
\]

5) \[
\frac{8}{16} = \frac{1}{2}
\]

Adding and Subtracting Fractions with the Different Denominators

Step 1: Since the fractions have different denominators find the Lowest Common Denominator.

Step 2: Replace any fraction as necessary with an equivalent fraction so that all fractions have the same denominator.

Step 3: Add or subtract the numerators.

Step 4: Simplify the fraction if necessary.
Example 8: Subtract $\frac{5}{6} - \frac{1}{3}$.

\[
\begin{array}{c|c|c}
\frac{5}{6} & \frac{1}{3} & \text{Subtract.} \\
6 & 3 & \text{Notice the denominators are not the same.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
6 \rightarrow 6, 12, 18, 24 \text{ and } 3 \rightarrow 3, 6, 9, 12, 15 & \text{The LCD is 6.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\frac{5}{6} \Rightarrow \frac{5}{6} & \text{Create equivalent fractions using the LCD.} \\
\frac{1}{3} \Rightarrow \frac{2}{6} & \text{The denominators should be the same.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\frac{5}{6} - \frac{2}{6} & \text{Subtract the numerators.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\frac{3}{6} & \text{Simplify the fraction.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\frac{3}{6} \Rightarrow \frac{1}{2} & \text{Divide the numerator and denominator by 3.} \\
\end{array}
\]

Example 9: Add $\frac{2}{5} + \frac{3}{7}$.

\[
\begin{array}{c|c|c}
\frac{2}{5} & \frac{3}{7} & \text{Add.} \\
5 \rightarrow 5, 10, 15, 20, 25, 30, 35 \text{ and } 7 \rightarrow 7, 14, 21, 28, 35 & \text{Notice the denominators are not the same.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
2 \rightarrow 2, 7 & \text{The LCD is 35.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\frac{2}{5} \Rightarrow \frac{14}{35} & \text{Create equivalent fractions using the LCD.} \\
\frac{3}{7} \Rightarrow \frac{15}{35} & \text{The denominators should be the same.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\frac{14}{35} & \text{Add the numerators.} \\
\frac{15}{35} & \text{The only common factor is one so the fraction is simplified.} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\frac{29}{35} & \text{} \\
\end{array}
\]
Try these exercises.

6) \(\frac{7}{8} - \frac{2}{4}\)  
11) \(\frac{2}{3} + \frac{1}{7}\)

7) \(\frac{3}{7} - \frac{1}{14}\)  
12) \(\frac{5}{8} + \frac{3}{20}\)

8) \(\frac{7}{18} - \frac{2}{9}\)  
13) \(\frac{7}{12} + \frac{1}{3}\)

9) \(\frac{5}{18} - \frac{2}{45}\)  
14) \(\frac{2}{3} + \frac{1}{4}\)

10) \(\frac{11}{24} - \frac{1}{6}\)  
15) \(\frac{1}{3} + \frac{1}{9} + \frac{2}{27}\)

Answers:

6) \(\frac{3}{8}\)  
10) \(\frac{7}{24}\)  
14) \(\frac{11}{12}\)

7) \(\frac{5}{14}\)  
11) \(\frac{17}{21}\)  
15) \(\frac{14}{27}\)

8) \(\frac{3}{18} = \frac{1}{6}\)  
12) \(\frac{31}{40}\)

9) \(\frac{21}{90} = \frac{7}{30}\)  
13) \(\frac{11}{12}\)