Fractions

Purpose
One of the areas most frustrating for teachers and students alike is the study of fractions, specifically operations with fractions. Year after year, students learn and forget how to add, subtract, multiply and divide with fractions. This handout will hopefully bring clarity to operations with fractions, since it is such an integral part of all adult mathematics courses.

Addition of Fractions

1. To add (or subtract) fractions, the denominators must be equal!
   a. Build each fraction (if needed) so that both denominators are equal.
   b. Add the numerators of the fractions.
   c. The new denominator will be the denominator of the built-up fractions. Reduce or simplify your answer, if needed.

<table>
<thead>
<tr>
<th>Addition</th>
<th>Procedure</th>
<th>Answer</th>
</tr>
</thead>
</table>
| \( \frac{2}{3} + \frac{7}{16} \) | 1. Lowest Common Denominator=48 (3 x 16)  
2. Build equivalent fractions:  
   \( \frac{2}{3}=\frac{32}{48}; \frac{7}{16}=\frac{21}{48} \) | \( \frac{32}{48} + \frac{21}{48} = \frac{53}{48} = 1 \frac{5}{48} \) |
| \( \frac{3}{8} + \frac{7}{16} \) | 1. Lowest Common Denominator=16 (Both 8 and 16 go evenly into 16)  
2. Build equivalent fractions:  
   \( \frac{3}{8}=\frac{6}{16}; \frac{7}{16}=\frac{7}{16} \) | \( \frac{6}{16} + \frac{7}{16} = \frac{13}{16} \) |
### Subtraction

To subtract fractions, the denominators must be equal. You basically follow the same steps as in addition (review previous section if necessary).

- **a.** Build each fraction (if needed) so that both denominators are equal.
- **b.** Subtract the numerators of the fractions.
- **c.** Reduce or simplify your answer, if needed.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Procedure</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{9} + \frac{2}{9} )</td>
<td>1. Denominators are the same, so there is no need to build equivalent fractions</td>
<td>( \frac{5}{9} + \frac{2}{9} = \frac{7}{9} )</td>
</tr>
<tr>
<td>( \frac{7}{8} + \frac{5}{12} )</td>
<td>1. Lowest Common Denominator=24 (Lowest number that can be divided evenly by both 8 and 12) 2. Build equivalent fractions: ( \frac{21}{24} + \frac{10}{24} = \frac{31}{24} )</td>
<td>( \frac{7}{8} = \frac{21}{24}; \frac{5}{12} = \frac{10}{24} )</td>
</tr>
<tr>
<td>( \frac{2}{3} - \frac{7}{16} )</td>
<td>1. Lowest Common Denominator=48 2. Build equivalent fractions: ( \frac{2}{3} = \frac{32}{48}; \frac{7}{16} = \frac{21}{48} )</td>
<td>( \frac{32}{48} - \frac{21}{48} = \frac{11}{48} )</td>
</tr>
<tr>
<td>( \frac{5}{9} - \frac{2}{9} )</td>
<td>1. Denominators are the same, so there is no need to build equivalent fractions</td>
<td>( \frac{5}{9} - \frac{2}{9} = \frac{3}{9} = \frac{1}{3} )</td>
</tr>
<tr>
<td>( \frac{7}{8} - \frac{5}{12} )</td>
<td>1. Lowest Common Denominator=24 2. Build equivalent fractions: ( \frac{7}{8} = \frac{21}{24}; \frac{5}{12} = \frac{10}{24} )</td>
<td>( \frac{21}{24} - \frac{10}{24} = \frac{11}{24} )</td>
</tr>
</tbody>
</table>
Multiplication

1. To multiply two proper fractions, complete the following:
   a. Multiply the numerators.
   b. Multiply the denominators
   c. Reduce or simplify your answer, if needed.

2. To multiply a whole number and a fraction, complete the following steps.
   a. Convert the whole number to a fraction.
   b. Multiply the numerators.
   c. Multiply the denominators
   d. Reduce or simplify your answer, if needed.

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Procedure</th>
<th>Answer</th>
</tr>
</thead>
</table>
| \( \frac{2}{3} \times \frac{7}{16} \) | 1. Multiply the numerators.  
2. Multiply the denominators | \( \frac{2 \times 7}{3 \times 16} = \frac{14}{48} = \frac{7}{24} \) |
| \( \frac{5}{9} \times \frac{2}{9} \) | 1. Multiply the numerators.  
2. Multiply the denominators | \( \frac{5 \times 2}{9 \times 9} = \frac{10}{81} \) |
| \( \frac{3}{8} \times \frac{5}{16} \) | 1. Multiply the numerators.  
2. Multiply the denominators | \( \frac{3 \times 5}{8 \times 16} = \frac{15}{128} \) |
| \( 3 \times \frac{5}{16} \) | 1. Convert the whole number to a fraction  
2. Multiply the numerators.  
3. Multiply the denominators | \( \frac{3 \times 5}{1 \times 16} = \frac{15}{16} \) |
### Division

1. **To divide one fraction by a second fraction, convert the problem to multiplication and multiply the two fractions.**
   a. Invert the fraction in the denominator
   b. Multiply the numerators.
   c. Multiply the denominators
   d. Reduce or simplify your answer, if needed.

2. **To divide one fraction by a whole number, convert the problem to multiplication and multiply the two fractions.**
   a. Convert the whole number to a fraction.
   b. Invert the fraction in the denominator.
   c. Multiply the numerators.
   d. Multiply the denominators
   e. Reduce or simplify your answer, if needed.
<table>
<thead>
<tr>
<th>Division</th>
<th>Procedure</th>
<th>Answer</th>
</tr>
</thead>
</table>
| \( \frac{2}{3} \) \( \frac{7}{16} \) | 1. Invert the fraction in the denominator 2. Multiply the numerators. 3. Multiply the denominators | \( \frac{7}{16} \rightarrow \frac{16}{7} \) 
\( \frac{16 \times 2}{7 \times 3} = \frac{32}{21} = \frac{11}{21} \) |
| \( \frac{5}{9} \) \( \frac{2}{9} \) | 1. Invert the fraction in the denominator 2. Multiply the numerators. 3. Multiply the denominators | \( \frac{2}{9} \rightarrow \frac{9}{2} \) 
\( \frac{9 \times 5}{2 \times 9} = \frac{45}{18} = \frac{9}{18} = \frac{1}{2} \) |
| \( \frac{7}{8} \) \( \frac{5}{5} \) | 1. Convert the whole number to a fraction 2. Invert the fraction in the denominator 3. Multiply the numerators. 4. Multiply the denominators | \( 5 \rightarrow \frac{5}{1} \) 
\( \frac{1 \times 7}{5 \times 8} = \frac{7}{40} \) |

**Five Common Mistakes to Avoid When Working with Fractions.**

1. **Adding two fractions without using a common denominator.**

   Example:

   \[
   \frac{2}{3} + \frac{4}{5} \neq \frac{2+4}{3+5}
   \]

   If the two fractions we are trying to add don’t have the same denominator, then we must rewrite each one as an equivalent fraction with a common denominator. We never add denominators when adding fractions. The correct answer is \( \frac{22}{15} \)

   Example:

   \[
   \frac{3}{5} + \frac{2}{3} \neq \frac{3+2}{5+3} \neq \frac{2}{5}
   \]

   crossing 3 from the numerator and denominator.

   The mistake is the denominators are added and the 3’s are crossed out.

   The correct answer is \( \frac{19}{15} \).
2. When multiplying two fractions, we do not need a common denominator.

3. When dealing with division of fractions, we multiply by the reciprocal of the top fraction instead of the reciprocal of the bottom fraction (divisor).

Example:
\[
\frac{2}{3} \div \frac{5}{6} \neq \frac{3}{2} \div \frac{5}{6}
\]
Remember we perform division by multiplying by the reciprocal of the divisor (the fraction to the right of the division symbol). The correct form is
\[
\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} = \frac{4}{5}
\]

4. When working with mixed numbers do not confuse mixed number notation for multiplication of fractions.

Example:
\[
3 \frac{2}{5} \text{ does not mean } 3 \times \frac{2}{5}; \text{ it means } 3 + \frac{2}{5}
\]

Example:
\[
-3 \frac{2}{5} \text{ does not mean } -3 + \frac{2}{5}; \text{ it means } -3 - \frac{2}{5}
\]

5. Subtraction with mixed numbers. We should change the mixed number to an improper fraction before subtracting.

Example:
Evaluate \(25 \frac{1}{7} - 4 \frac{3}{7}\)

Step 1: \(25 \frac{1}{7} = 7 \times 25 + 1 = 176\)

Step 2: \(4 \frac{3}{7} = 7 \times 4 + 3 = 31\)

Step 3: \(176 - 31 = 145 = 205\)
Practice Test

1. Reduce \( \dfrac{21}{238} \) to simplest form

A) \( \dfrac{7}{79} \)  B) \( \dfrac{3}{34} \)  C) \( \dfrac{1}{11} \)  D) \( \dfrac{7}{34} \)

2. Add: \( \dfrac{3}{8} + \dfrac{5}{4} \)

A) \( \dfrac{8}{12} \)  B) \( \dfrac{2}{3} \)  C) \( \dfrac{15}{8} \)  D) \( \dfrac{13}{4} \)

3. Add: \( 4\dfrac{1}{5} + 2\dfrac{1}{3} \)

A) \( 6\dfrac{2}{15} \)  B) \( 6\dfrac{2}{5} \)  C) \( 6\dfrac{1}{4} \)  D) \( 6\dfrac{8}{15} \)

4. Subtract and leave as a mixed number: \( 3\dfrac{1}{2} - 1\dfrac{2}{3} \)

A) \( \dfrac{11}{6} \)  B) \( 2\dfrac{1}{3} \)  C) \( 2\dfrac{1}{6} \)  D) \( 1\dfrac{5}{6} \)

5. Multiply (and reduce if possible): \( \dfrac{13}{25} \times \dfrac{40}{26} \)

A) \( \dfrac{4}{5} \)  B) \( \dfrac{5}{6} \)  C) \( 1\dfrac{2}{7} \)  D) \( \dfrac{169}{500} \)

6. Multiply (and reduce if possible): \( 3\dfrac{3}{25} \times 2\dfrac{1}{2} \)

A) \( 7\dfrac{4}{5} \)  B) \( 6\dfrac{3}{5} \)  C) \( 6\dfrac{3}{50} \)  D) \( 5\dfrac{3}{50} \)

7. Divide (and reduce if possible): \( \dfrac{5}{8} \div \dfrac{3}{4} \)

A) \( \dfrac{6}{5} \)  B) \( 15\dfrac{15}{32} \)  C) \( \dfrac{5}{6} \)  D) \( \dfrac{32}{15} \)

8. Divide (and reduce if possible): \( 1\dfrac{1}{5} \div 2\dfrac{1}{10} \)

A) \( \dfrac{4}{7} \)  B) \( \dfrac{25}{63} \)  C) \( 1\dfrac{3}{4} \)  D) \( 2\dfrac{13}{25} \)
9. A bank processed 176 transactions on Thursday and 256 transactions on Friday. Find the ratio, as a fraction in simplest form, of the number of transactions on Thursday to the number of transactions on Friday.

A) \( \frac{16}{11} \) \hspace{1cm} B) \( \frac{176}{256} \) \hspace{1cm} C) \( \frac{88}{128} \) \hspace{1cm} D) \( \frac{11}{16} \)

10. Simplify: \( \frac{7}{10} + \frac{7}{15} \)

A) \( \frac{14}{25} \) \hspace{1cm} B) \( \frac{7}{30} \) \hspace{1cm} C) \( \frac{7}{12} \) \hspace{1cm} D) \( 1\frac{1}{6} \)

11. Simplify: \( -\frac{3}{8} \left(-\frac{2}{7}\right) \)

A) \( -\frac{5}{56} \) \hspace{1cm} B) \( -\frac{1}{3} \) \hspace{1cm} C) \( \frac{5}{56} \) \hspace{1cm} D) \( -\frac{1}{15} \)

12. Divide: \( -13\frac{1}{15} ÷ -4\frac{9}{10} \)

A) \( \frac{1}{3} \) \hspace{1cm} B) \( 2\frac{2}{3} \) \hspace{1cm} C) \( -3\frac{2}{3} \) \hspace{1cm} D) \( 3\frac{2}{3} \)

13. Divide: \( 8 ÷ -3\frac{1}{5} \)

A) \( -1\frac{3}{5} \) \hspace{1cm} B) \( 2\frac{1}{2} \) \hspace{1cm} C) \( -25\frac{3}{5} \) \hspace{1cm} D) \( -2\frac{1}{2} \)

Practice Test Answers

1. B \hspace{1cm} 2. C \hspace{1cm} 3. D \hspace{1cm} 4. D \hspace{1cm} 5. A \hspace{1cm} 6. A \hspace{1cm} 7. C \hspace{1cm} 8. A \hspace{1cm} 9. D \hspace{1cm} 10. D \hspace{1cm} 11. A \hspace{1cm} 12. B \hspace{1cm} 13. D