**Objective:**

This handout will help students understand prime numbers, composite numbers, prime factors and divisibility rules.

**Vocabulary Review:**

*Factor* - a number that is being multiplied to get another number, the product.

*Prime number* - a number that has exactly two factors, 1 and itself. Prime numbers are the building blocks of all numbers. The number 1 is not a prime number because it has only one factor.

*Composite number* – a number that has more factors than 1 and itself. Composite numbers are made up of prime numbers. The word “composite” means to combine things.

*Prime factorization* – when only prime numbers are multiplied together to get a product.

<table>
<thead>
<tr>
<th>Number</th>
<th>Factorization</th>
<th>Factors</th>
<th>Prime or Composite?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 x 1</td>
<td>1</td>
<td>Neither</td>
</tr>
<tr>
<td>2</td>
<td>1 x 2</td>
<td>1, 2</td>
<td>Prime</td>
</tr>
<tr>
<td>3</td>
<td>1 x 3</td>
<td>1, 3</td>
<td>Prime</td>
</tr>
<tr>
<td>4</td>
<td>1 x 4 ; 2 x 2</td>
<td>1, 2, 4</td>
<td>Composite</td>
</tr>
<tr>
<td>5</td>
<td>1 x 5</td>
<td>1, 5</td>
<td>Prime</td>
</tr>
<tr>
<td>6</td>
<td>1 x 6 ; 2 x 3</td>
<td>1, 2, 3, 6</td>
<td>Composite</td>
</tr>
<tr>
<td>7</td>
<td>1 x 7</td>
<td>1, 7</td>
<td>Prime</td>
</tr>
<tr>
<td>8</td>
<td>1 x 8 ; 2 x 4</td>
<td>1, 2, 4, 8</td>
<td>Composite</td>
</tr>
<tr>
<td>9</td>
<td>1 x 9 ; 3 x 3</td>
<td>1, 3, 9</td>
<td>Composite</td>
</tr>
<tr>
<td>10</td>
<td>1 x 10 ; 2 x 5</td>
<td>1, 2, 5, 10</td>
<td>Composite</td>
</tr>
<tr>
<td>11</td>
<td>1 x 11</td>
<td>1, 11</td>
<td>Prime</td>
</tr>
<tr>
<td>12</td>
<td>1 x 12 ; 2 x 6 ; 3 x 4</td>
<td>1, 2, 3, 4, 6, 12</td>
<td>Composite</td>
</tr>
<tr>
<td>13</td>
<td>1 x 13</td>
<td>1, 13</td>
<td>Prime</td>
</tr>
<tr>
<td>14</td>
<td>1 x 14 ; 2 x 7</td>
<td>1, 2, 7, 14</td>
<td>Composite</td>
</tr>
<tr>
<td>15</td>
<td>1 x 15 ; 3 x 5</td>
<td>1, 3, 5, 15</td>
<td>Composite</td>
</tr>
<tr>
<td>16</td>
<td>1 x 16 ; 2 x 8 ; 4 x 4</td>
<td>1, 2, 4, 8, 16</td>
<td>Composite</td>
</tr>
<tr>
<td>17</td>
<td>1 x 17</td>
<td>1, 17</td>
<td>Prime</td>
</tr>
</tbody>
</table>

What do you think the biggest prime number is? Is it over a 100? Over 1,000? Check the Internet and see if you guessed correctly. In January 2013 a record was set for the largest prime number.
Prime factorization

A factor tree is one technique that can be used to find the prime factorization of a number.

**Example 1**: Find the prime factorization of 42.

```
2 x 7 = 42
/   /   \
2   7   6
/ / \
2 3
```

**Example 2**: Find the prime factorization of 48.

```
2 x 3 x 2 x 2 x 2 = 48
/   /   /   /   \\
2   3   2   2   4
/ /  /  /  /  \\  \  \\
2 3  2  2  2
```

**Try these exercises**:

Find the prime factorization of the following numbers:

1) 75  
2) 20  
3) 126  
4) 225  
5) 68

**Answers**:

1) \(3 \times 5 \times 5 \iff 3 \cdot 5^2\)
2) \(2 \times 2 \times 5 \iff 2^2 \cdot 5\)
3) \(2 \times 3 \times 3 \times 7 \iff 2 \cdot 3^2 \cdot 7\)
4) \(3 \times 3 \times 5 \times 5 \iff 3^2 \cdot 5^2\)
5) \(2 \times 2 \times 17 \iff 2^2 \cdot 17\)
Divisibility
Divisibility - capable of evenly dividing one number by another number, without a remainder. The result is a whole number.

- 14 is divisible by 7, because $14 \div 7 = 2$ exactly

- 15 is not divisible by 7, because $15 \div 7 = 2 R.1$ or $2 \frac{1}{7}$ (The result is not a whole number)

Rules for Divisibility allow you to quickly determine if a number is divisible by another number.

**Divisibility by 2:** if the last digit of a number is even. It ends with a 0, 2, 4, 6, or 8.

   *Example:* 208 ends with an “8” so it is divisible by 2.

**Divisibility by 3:** if the sum of the digits of a number are divisible by 3

   *Example:* 3321 ⇒ $3+3+2+1 = 9$ and 9 is divisible by 3. Thus 3321 is divisible by 3.

**Divisibility by 4:** if the last two digits of a number are divisible by 4

   *Example:* 8924 ⇒ 24 is divisible by 4, thus 8924 is divisible by 4.

**Divisibility by 5:** if the last digit of a number is “0” or “5”.

   *Example:* 4235 ends with a “5”, thus 4235 is divisible by 5.

**Divisibility by 6:** if a number is divisible by both 2 and 3. (Use the divisibility rules for 2 and 3 both: if the number is an even number and the sum of the digits are divisible by 3)

   *Example:* 5322 is an even number and $5+3+2+2=12$ and 12 is divisible by 3. Thus 5322 is divisible by 6.

**Divisibility by 9:** if the sum of the digits of a number is divisible by 9

   *Example:* 3141 ⇒ $3+1+4+1=9$ and 9 is divisible by 9, thus 3141 is divisible by 9.

**Divisibility by 10:** if the last digit of a number is “0”

   *Example:* 630 ends in “0”, thus 630 is divisible by 10.
Example 3:

Use the divisibility rules to decide whether the number 2,324 is divisible by 2, 3, 4, 5, 6, 9 and 10.

The number 2324 is divisible by 2 and 4. The number ends in 4 (even number) and the last two digits (24) are divisible by 4.

Try these exercises:

Use the divisibility rules to determine whether the following numbers are divisible by 2, 3, 4, 5, 6, 9 and 10.

1) 273  2) 391  3) 4,164  4) 5,940  5) 2,805

<table>
<thead>
<tr>
<th>Answers:</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 273</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) 391</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) 4,164</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4) 5,940</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>5) 2,805</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
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