Trigonometry Basic Practice Problems

Do the following for problems 1 through 3:

a) Draw and label the given angle,

b) Find a positive co-terminal angle less than one revolution, then draw and label it,

c) Find the reference angle, then draw and label it.

1. $\theta = 550^\circ$  
2. $\theta = \frac{11\pi}{3}$  
3. $\theta = -210^\circ$

4. Convert $25^\circ 17' 39''$ to decimal degrees. (Round to 3 decimal places.)

5. a) What do you multiply a degree angle by to convert to radians?

b) What do you multiply a radian angle by to convert to degrees?

6. Change the radian measure to degree measure in exact form.

   a) $\frac{\pi}{12}$  
   b) $-\frac{5\pi}{4}$  
   c) 3

7. Change the degree measure to radian measure in exact form.

   a) $330^\circ$  
   b) $-225^\circ$  
   c) $140^\circ$

8. By memory, state the arc length formula associated with the angle $\theta$ measured in

   a) degrees  
   b) radians

9. If Chicago is located at $42^\circ$ N latitude and $88^\circ$ W longitude, what is the approximate distance (to the nearest mile) from the equator? Assume the diameter of the earth is 7,900 miles.

10. By memory, state the formulas for

    a) angular velocity  
    b) linear velocity
11. A belt runs a pulley of radius 10 inches at 50 rev/min. Find the angular velocity (in rad/hr) of the pulley and the linear velocity (in inches/sec) of the belt.

12. By memory, complete the identities.

   a) \( \tan \theta = \frac{1}{\cdots} \)  
   b) \( \csc \theta = \frac{1}{\cdots} \)
   c) \( \sec \theta = \frac{1}{\cdots} \)  
   d) \( \cdots = \frac{\cos \theta}{\sin \theta} \)
   e) \( \cdots = \frac{\sin \theta}{\cos \theta} \)  
   f) \( \cos^2 \theta + \cdots = \cdots \)
   g) \( \cdots + \tan^2 \theta = \cdots \)  
   h) \( \cot^2 \theta + \cdots = \cdots \)

13. Manipulate the above identities to fill in the blanks.

   a) \( \cos \theta \tan \theta = \cdots \)  
   b) \( \sin \theta = \frac{1}{\cdots} \)
   c) \( 1 - \sin^2 \theta = \cdots \)  
   d) \( \csc^2 \theta - \cot^2 \theta = \cdots \)

14. A ray from the origin through the point \((-3, -1)\) forms an angle \(\theta\) with the x-axis. Find the following.

   a) \( \cos \theta \)  
   b) \( \sin \theta \)  
   c) \( \tan \theta \)
   d) \( \cot \theta \)  
   e) \( \sec \theta \)  
   f) \( \csc \theta \)

15. By memory, fill in the table with exact values.

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<tr>
<th>(\theta)</th>
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16. Give the exact values of each.

a) \( \cos 225^\circ \)  
   b) \( \csc 330^\circ \)  
   c) \( \tan (-300^\circ) \)

   d) \( \cot (-90^\circ) \)  
   e) \( \sin 120^\circ \)  
   f) \( \sec 405^\circ \)

   g) \( \sin \frac{11\pi}{6} \)  
   h) \( \tan \frac{3\pi}{4} \)  
   i) \( \cot \left( -\frac{2\pi}{3} \right) \)

   j) \( \sec \left( -\frac{11\pi}{6} \right) \)  
   k) \( \cos 3\pi \)  
   l) \( \csc \frac{15\pi}{4} \)

17. Use the calculator to evaluate. (Round to two decimal places.)

a) \( \cos 40^\circ \)  
   b) \( \csc 125^\circ \)  
   c) \( \cot (-20^\circ) \)

   d) \( \sec^2 \left( -\frac{5\pi}{12} \right) \)  
   e) \( \sin^2 \left( \frac{\pi}{4} \right) + \cos^2 \left( \frac{\pi}{4} \right) \)

Solutions

Do the following for problems 1 through 3:

a) Draw and label the given angle,

b) Find a positive co-terminal angle less than one revolution, then draw and label it,

c) Find the reference angle, then draw and label it.

1. \( \theta = 550^\circ \)

   co-terminal angle: \( 550^\circ - 360^\circ = 190^\circ \)

   reference angle: \( 10^\circ \)

2. \( \theta = \frac{11\pi}{3} \)

   co-terminal angle: \( \frac{11\pi}{3} - 2\pi = \frac{5\pi}{3} \)

   reference angle: \( \frac{\pi}{3} \)
3. \( \theta = -210^\circ \)

   co-terminal angle: \(-210 + 360 = 150^\circ\)
   reference angle: \(30^\circ\)

4. Convert \(25^\circ 17' 39''\) to decimal degrees. (Round to 3 decimal places.)

   \[25^\circ 17' 39'' = 25^\circ + \left(\frac{17}{60}\right)^\circ + \left(\frac{39}{3600}\right)^\circ = 25.294^\circ\]

5. a) What do you multiply a degree angle by to convert to radians?

   \[\frac{\pi}{180}\]

   b) What do you multiply a radian angle by to convert to degrees?

   \[\frac{180}{\pi}\]

6. Change the radian measure to degree measure in exact form.

   a) \[\frac{\pi}{12} \cdot \frac{180}{\pi} = \frac{180}{12} = 15^\circ\]

   b) \[-\frac{5\pi}{4} \cdot \frac{180}{\pi} = -\frac{900}{4} = -225^\circ\]

   c) \[3 \cdot \frac{180}{\pi} = \frac{540^\circ}{\pi}\]

7. Change the degree measure to radian measure in exact form.

   a) \[330^\circ \cdot \frac{\pi}{180} = \frac{330\pi}{180} = \frac{11\pi}{6}\text{ radians}\]

   b) \[-225^\circ \cdot \frac{\pi}{180} = -\frac{225\pi}{180} = -\frac{5\pi}{4}\text{ radians}\]

   c) \[140^\circ \cdot \frac{\pi}{180} = \frac{140\pi}{180} = \frac{7\pi}{9}\text{ radians}\]

8. By memory, state the arc length formula associated with the angle \(\theta\) measured in

   a) degrees: \(s = \frac{\theta}{360} (2\pi r)\)  

   b) radians: \(s = r\theta\)
9. If Chicago is located at 42° N latitude and 88° W longitude, what is the approximate distance (to the nearest mile) from the equator? Assume the diameter of the earth is 7,900 miles.

\[ \theta = 42° \quad \text{diameter: 7900 mi} \quad \therefore \text{radius } r = 3950 \text{ mi} \]

\[ s = \frac{\theta}{360} \left(2\pi \cdot r\right) = \frac{2765\pi}{3} \approx 2896 \text{ mi to the equator} \]

10. By memory, state the formulas for

a) angular velocity: \( \omega = \frac{\theta}{t} \)

b) linear velocity: \( v = r\omega \)

11. A belt runs a pulley of radius 10 inches at 50 rev/min. Find the angular velocity (in rad/hr) of the pulley and the linear velocity (in inches/sec) of the belt.

angular velocity: \( \omega = \frac{50 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ radians}}{\text{rev}} \cdot \frac{60 \text{ min}}{\text{hr}} = \frac{6000\pi \text{ radians}}{\text{hr}} \approx 18849.56 \text{ rad/hr} \)

linear velocity: \( v = r\omega = \frac{10 \text{ in}}{\text{rad}} \cdot \frac{6000\pi \text{ rad}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} = \frac{50\pi \text{ in}}{3 \text{ sec}} \approx 52.36 \text{ in/sec} \)

12. By memory, complete the identities.

a) \( \tan \theta = \frac{1}{\cot \theta} \)

b) \( \csc \theta = \frac{1}{\sin \theta} \)

c) \( \sec \theta = \frac{1}{\cos \theta} \)

d) \( \cot \theta = \frac{\cos \theta}{\sin \theta} \)

e) \( \tan \theta = \frac{\sin \theta}{\cos \theta} \)

f) \( \cos^2 \theta + \sin^2 \theta = 1 \)

g) \( 1 + \tan^2 \theta = \sec^2 \theta \)

h) \( \cot^2 \theta + 1 = \csc^2 \theta \)

13. Manipulate the above identities to fill in the blanks.

a) \( \cos \theta \tan \theta = \sin \theta \)

b) \( \sin \theta = \frac{1}{\csc \theta} \)

c) \( 1 - \sin^2 \theta = \cos^2 \theta \)

d) \( \csc^2 \theta - \cot^2 \theta = 1 \)
14. A ray from the origin through the point (-3, -1) forms an angle \( \theta \) with the x-axis. Find the following.

\[ x = -3 \quad y = -1 \quad r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10} \]

a) \( \cos \theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10} \)

b) \( \sin \theta = \frac{y}{r} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10} \)

c) \( \tan \theta = \frac{y}{x} = \frac{-1}{-3} = \frac{1}{3} \)

d) \( \cot \theta = \frac{x}{y} = \frac{-3}{-1} = 3 \)

e) \( \sec \theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3} \)

f) \( \csc \theta = \frac{r}{y} = \frac{\sqrt{10}}{-1} = -\sqrt{10} \)

15. By memory, fill in the table with exact values.

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<tr>
<th>( \theta )</th>
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<th>( \cos \theta )</th>
<th>( \sin \theta )</th>
<th>( \tan \theta )</th>
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16. Give the exact values of each.

a) \( \cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2} \)

b) \( \csc 330° = \frac{1}{\sin 330°} = \frac{1}{-\sin 30°} = \frac{1}{-\frac{1}{2}} = -2 \)

c) \( \tan(-300°) = \tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\sqrt{3}}{\frac{1}{2}} = \sqrt{3} \)

d) \( \cot(-90°) = \frac{\cos (-90°)}{\sin (-90°)} = \frac{0}{-1} = 0 \)

e) \( \sin 120° = \sin 60° = \frac{\sqrt{3}}{2} \)

f) \( \sec 405° = \sec 45° = \frac{1}{\cos 45°} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} \)

g) \( \sin \frac{11\pi}{6} = -\sin \frac{\pi}{6} = -\frac{1}{2} \)

h) \( \tan \frac{3\pi}{4} = -\tan \frac{\pi}{4} = -1 \)
i) \[ \cot \left( -\frac{2\pi}{3} \right) = \cot \frac{\pi}{3} = \frac{\cos \frac{\pi}{3}}{\sin \frac{\pi}{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \]

j) \[ \sec \left( -\frac{11\pi}{6} \right) = \sec \frac{\pi}{6} = \frac{1}{\cos \frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \]

k) \[ \cos 3\pi = \cos \pi = -1 \]

l) \[ \csc \frac{15\pi}{4} = -\csc \frac{\pi}{4} = -\frac{1}{\sin \frac{\pi}{4}} = -\frac{1}{\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} \]

17. Use the calculator to evaluate. (Round to two decimal places.)

a) \[ \cos 40^\circ \approx 0.77 \]

b) \[ \csc 125^\circ = \frac{1}{\sin 125^\circ} \approx 1.22 \]

c) \[ \cot(-20^\circ) = \frac{1}{\tan(-20^\circ)} \approx -2.75 \]

d) \[ \sec^2 \left( -\frac{5\pi}{12} \right) = \frac{1}{\cos^2 \left( -\frac{5\pi}{12} \right)} \approx 14.93 \]

e) \[ \sin^2 \left( \frac{\pi}{4} \right) + \cos^2 \left( \frac{\pi}{4} \right) = 1 \]